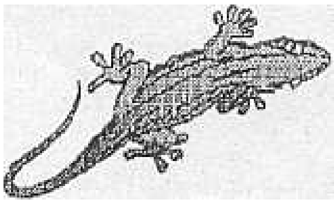
BACK

Johan Roeloffs

Didaktische Implikationen von Computeralgebra im Unterricht

I Erfahrungen mit 

Beispiele:

Einführung in die Integralrechnung

III Diskrete Faltung

f(i) mit $0 \le i \le m$ und g(j) mit $0 \le j \le n$

$$p(s) = \sum_{i=\max(0,s-n)}^{\min(s,m)} f(i)\cdot g(s-i)$$

IV stetige Faltung:

$$\text{Dichte}(s) = \int_{-\infty}^{\infty} f(x)\cdot g(s-x)\,dx$$

ein Ergebnis:

$$u_n(x) = \frac{n}{2}\sum_{k=0}^{n} \frac{(-1)^k\cdot sign(x-k)}{k!\cdot(n-k)!}\cdot(x-k)^{n-1}$$

VI Fragen

Introduction of Integral Calculation

"Utility's for Chap. 5: Integration.

INV(f,x):=ITERATE(f,x,x,-1

Y_(f,x,α):=ITERATE(f,x,α,1)

$$RI(f,x,a,b,n,\theta):=\frac{b-a}{n}\cdot\sum_{i=1}^{n} Y\_\left[f,x,a+\frac{(b-a)\cdot(i-1+\theta)}{n}\right]$$

S_LEFT(f,x,a,b,n):=RI(f,x,a,b,n,0)

S_RIGHT(f,x,a,b,n):=RI(f,x,a,b,n,1)

S_low(f,x,a,b,n):=IF(Y_(f,x,a)<Y_(f,x,b),RI(f,x,a,b,n,0), RI(f,x,a,b,n,1

S_UP(f,x,a,b,n):=IF(Y_(f,x,a)<Y_(f,x,b),RI(f,x,a,b,n,1), RI(f,x,a,b,n,0)

$$S\_MID(f,x,a,b,n):=RI\left[f,x,a,b,n,\frac{1}{2}\right]$$

$$S\_RANDOM(f,x,a,b,n):=\frac{b-a}{n}\cdot\sum_{i=1}^{n} Y\_\left[f,x,a+\frac{(b-a)\cdot(i-RANDOM(1))}{n}\right]$$

$$S\_TRAP(f,x,a,b,n):=\frac{b-a}{2\cdot n}\cdot\left[\left(\lim_{x\to a} f\right)+2\cdot\sum_{i=1}^{n-1} Y\_\left[f,x,a+\frac{b-a}{n}\cdot i\right]+\lim_{x\to b} f\right]$$

$$R\_TRAP(f,x,a,b,n):=-\frac{1}{12}\cdot\left[\frac{b-a}{n}\right]^2\cdot\left[\left[\lim_{x\to b}\frac{d}{dx} f\right]-\lim_{x\to a}\frac{d}{dx} f\right]$$

$$PART\_INT(f,g,x):=f\cdot\int g\,dx-\int\left(\int g\,dx\cdot\frac{d}{dx} f\right)dx$$

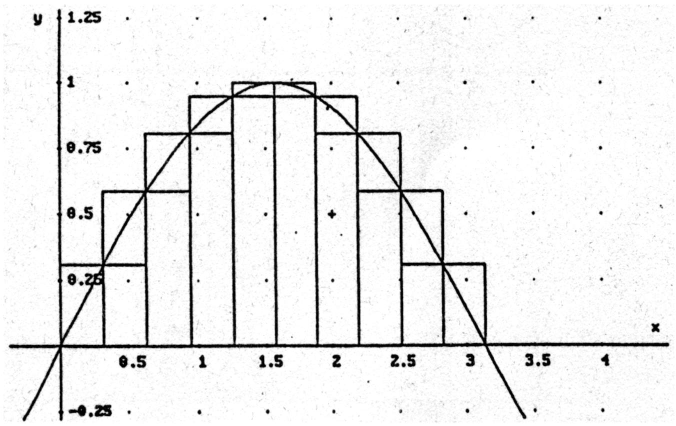
$$SUB\_INT(f,u,x):=Y\_\left[\int Y\_(f,x,INV(u,x))\cdot\frac{d}{dx} INV(u,x)\,dx,x,u\right]$$

$$SUB\_DINT(f,u,x,a,b):=\int_{Y\_(u,x,a)}^{Y\_(u,x,b)} Y\_(f,x,INV(u,x))\cdot\frac{d}{dx} INV(u,x)\,dx$$

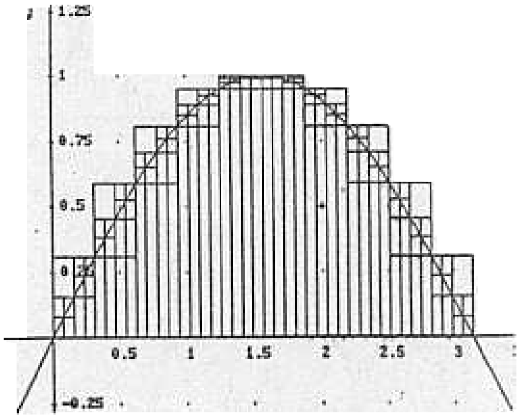
$$REV\_INT(f,x,a,guess,solution):=ITERATE\left[\frac{\int_a^x f\,dx-solution}{f}\right.$$

Riemannsom van sin(x) op $[0, \pi]$

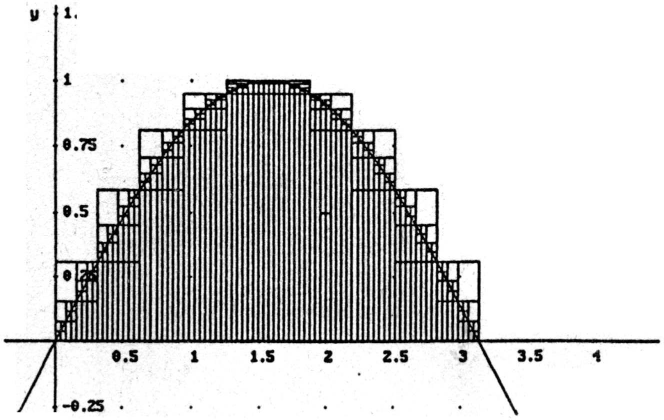
n = 10 steps

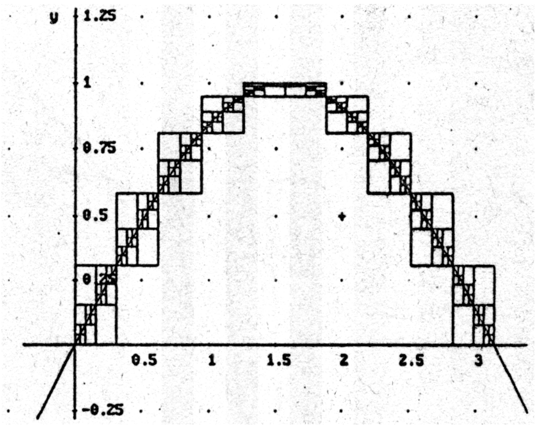


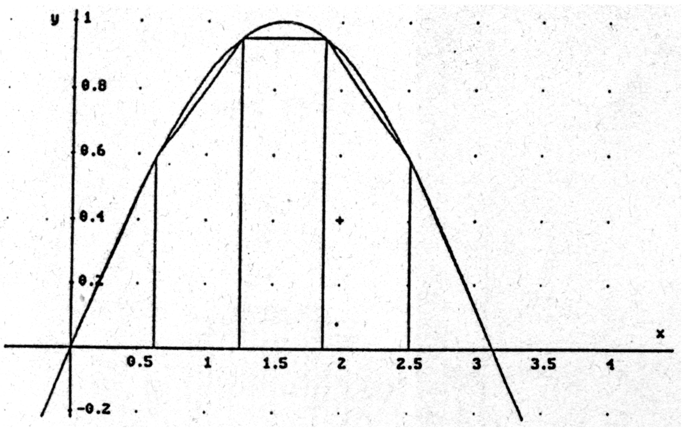
n = 40 steps

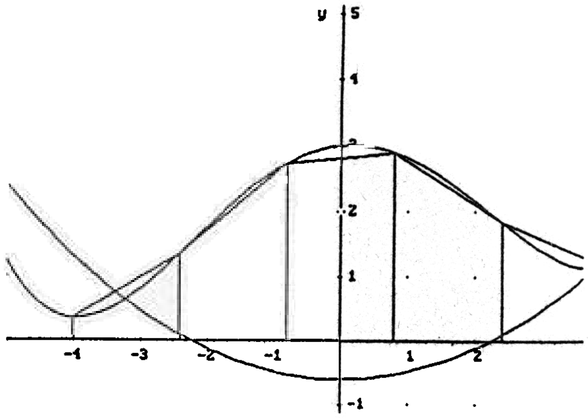


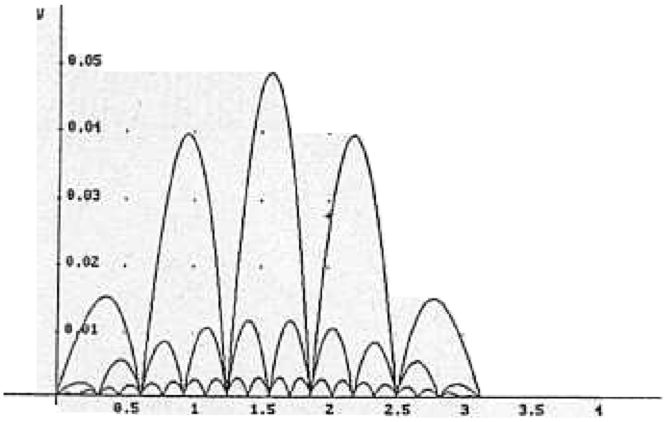
n = 80 steps

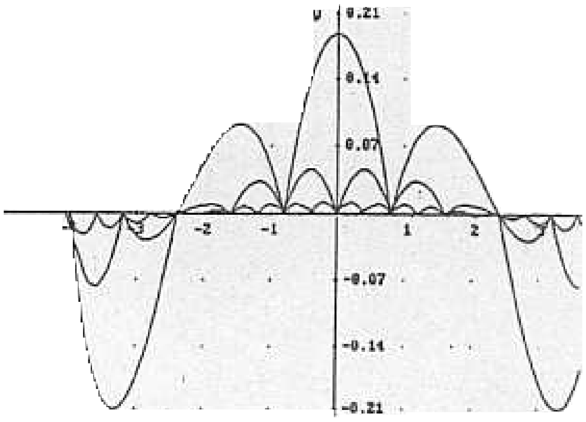


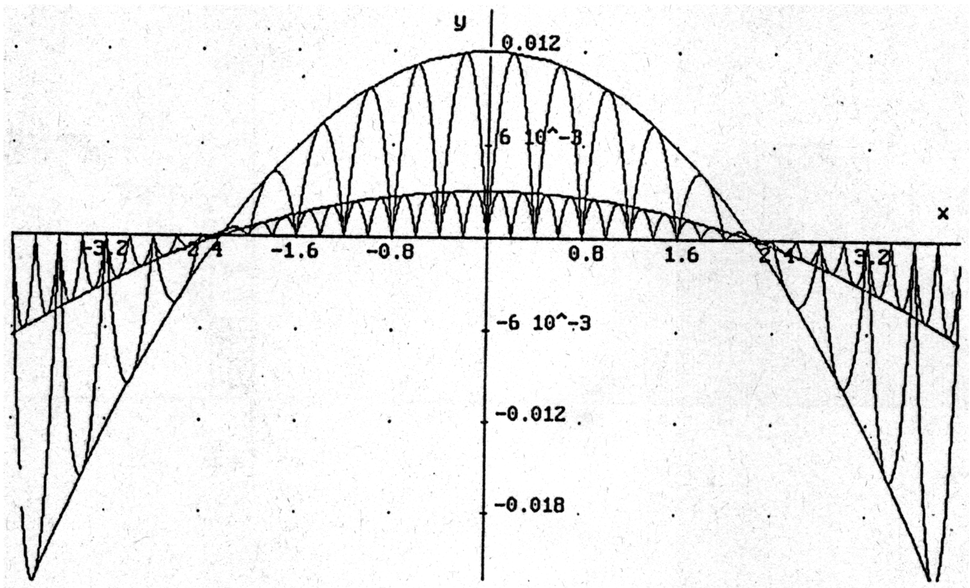



y
1
0.8
0.6
0.4
-0.2
x
0.5
1
1.5
2
2.5
3
3.5
4


y
5
2
1
-1
-4
-3
-1
1
2


y
0.05
0.04
0.03
0.02
0.5
1
1.5
2
2.5
3
3.5
4


0.21
0.07
-0.07
-0.14
-0.21
-2
-1
1
2


y
0.012
6 10^-3
x
-1.6
-0.8
0.8
1.6
-6 10^-3
-0.012
-0.018

Diskrete Faltung

"Functions for Statistics: Chap.'s 2,3,4 and 5.

$$E(f,k,m\_ax):=\sum_{i=0}^{m\_ax} i\cdot Y\_(f,k,i)$$

$$V(f,k,m\_ax):=\left[\sum_{i=0}^{m\_ax} i^2\cdot Y\_(f,k,i)\right]-E(f,k,m\_ax)^2$$

$$\sigma(f,k,m\_ax):=\sqrt{\left[\left[\sum_{i=0}^{m\_ax} i^2\cdot Y\_(f,k,i)\right]-E(f,k,m\_ax)^2\right]}$$

```
*****************************************************************
*                               MIN(s,m)                        *
*        CAUX(f,m,g,n,s) :=        Σ         f·g                *
*                              i=MAX(0,s-n)                     *
*                                                               *
*    CONV(f,m,g,n,s):=CAUX(Y_(f,k,i),m,Y_(g,k,s-i),n,s)         *
*****************************************************************
```

$$STAAF(f,k,m\_ax):=VECTOR\left[\begin{bmatrix} i & 0 \\ i & Y\_(f,k,i) \end{bmatrix},i,0,m\_ax\right]$$

$$HISTO(f,k,m\_ax):=VECTOR\left[\begin{bmatrix} i-\frac{1}{2} & 0 \\ i-\frac{1}{2} & Y\_(f,k,i) \\ i+\frac{1}{2} & Y\_(f,k,i) \\ i+\frac{1}{2} & 0 \end{bmatrix},i,0,m\_ax\right]$$

$$BER(p,k):=CHI\left[-\frac{1}{2},k,\frac{3}{2}\right]\ (2\cdot p-1)\cdot k+1-p)$$

$$BIN(n,p,k):=COMB(n,k)\cdot p^k\cdot(1-p)^{n-k}$$

$$BEU(n,p,x):=\frac{1}{\sqrt{(2\cdot\pi\cdot n\cdot p\cdot(1-p))}}\cdot ê^{-0.5\cdot(x-n\cdot p)^2/(n\cdot p\cdot(1-p)}$$

$$POI(\mu,k):=\frac{ê^{-\mu}\cdot\mu^k}{k!}$$

$$E(POI(\mu,k),k,\infty)=\mu$$

$$PEU(\mu,x):=\frac{1}{\sqrt{(2\cdot\pi\cdot\mu)}}\cdot ê^{-0.5\cdot(x-\mu)^2/\mu}$$

Von Bernoulli-Experiment zu Binomialverteilung

$$\text{VCTOR(CONV(BER(p),1,BER(p),1,s),s,0,2)} = \left[(p-1)^2, 2\cdot p\cdot(1-p), p\right.$$

$$\text{VECTOR(BIN(2, p), k, 0, 2)} = \left[(p-1)^2, 2\cdot p\cdot(1-p), p^2\right]$$

$$\text{VECTOR(CONV(BIN(2,p),2,BER(p),1,s),s,0,3)} =$$
$$\left[(1-p)^3, 3\cdot p\cdot(p-1^2, 3\cdot p^2\cdot(1-p), p^3\right]$$

$$\text{VECTOR(BIN(3, p),k, 0,3)} = \left[(1-p)^3, 3\cdot p\cdot(p-1^2, 3\cdot p^2\cdot(1-p), p\right.$$

$$\text{CONV(BIN(5,p),5,BIN(5,p),5,6)} = 210\cdot p^6\cdot(p-1^4$$

$$\text{BIN(10,p,6)} = 210\cdot p^6\cdot(p-1)^4$$

Der Würfler wirft mit Würfeln

De dobbelaar gooit met dobbelstenen

$$\text{D1(k)} := \frac{\text{CHI}\left[\frac{1}{2}, k, \frac{13}{2}\right]}{6}$$

$$\text{v1} := \left[0, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]$$

$$\text{m2} := \frac{1}{36}\cdot\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\text{v2} := \frac{1}{36}\cdot[0, 0, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1]$$

m3 := VECTOR(VECTOR(D2(k)·D1(r),k,0,12),r,0,6)

$\frac{1}{216}\cdot$

Som:				3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	2	3	4	5	6	5	4	3	2	1
0	0	1	2	3	4	5	6	5	4	3	2	1
0	0	1	2	3	4	5	6	5	4	3	2	1
0	0	1	2	3	4	5	6	5	4	3	2	1
0	0	1	2	3	4	5	6	5	4	3	2	1
0	0	1	2	3	4	5	6	5	4	3	2	1
Som:					12	13	14	15	16	17	18	

$$D3(i) := \sum_{k=MAX(0,i-6)}^{MIN(i,12)} m3_{i-k+1,k+1}$$

v3 := VECTOR(D3(k),k,0,18)

$$v3 := \frac{1}{216}\cdot[0,0,0,1,3,6,10,15,21,25,27,27,25,21,15,10,6,3,1]$$

m4 := VECTOR(VECTOR(D2(k)·D2(r),k,0,12),r,0,12)

$$\frac{1}{1296}\cdot\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 \\
0 & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 10 & 8 & 6 & 4 & 2 \\
0 & 0 & 3 & 6 & 9 & 12 & 15 & 18 & 15 & 12 & 9 & 6 & 3 \\
0 & 0 & 4 & 8 & 12 & 16 & 20 & 24 & 20 & 16 & 12 & 8 & 4 \\
0 & 0 & 5 & 10 & 15 & 20 & 25 & 30 & 25 & 20 & 15 & 10 & 5 \\
0 & 0 & 6 & 12 & 18 & 24 & 30 & 36 & 30 & 24 & 18 & 12 & 6 \\
0 & 0 & 5 & 10 & 15 & 20 & 25 & 30 & 25 & 20 & 15 & 10 & 5 \\
0 & 0 & 4 & 8 & 12 & 16 & 20 & 24 & 20 & 16 & 12 & 8 & 4 \\
0 & 0 & 3 & 6 & 9 & 12 & 15 & 18 & 15 & 12 & 9 & 6 & 3 \\
0 & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 10 & 8 & 6 & 4 & 2 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1
\end{bmatrix}$$

$$D4(i) := \sum_{k=MAX(0,i-12)}^{MIN(i,12)} m4_{i-k+1,k+1}$$

v4 := VECTOR(D4(k),k,0,24)

$$v4 = \frac{1}{1296} \cdot [0,0,0,0,1,4,10,20,35,56,80,104,125,140,146,140,125,104,80,56,35,20,10,4,1]$$

$$m6 := VECTOR(VECTOR(v3_k \cdot v3_r, k, 1, 19), r, 1, 19)$$

46656 · m6 =

$$\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&3&6&10&15&21&25&27&27&25&21&15&10&6&3&1\\
0&0&0&3&9&18&30&45&63&75&81&81&75&63&45&30&18&9&3\\
0&0&0&6&18&36&60&90&126&150&162&162&150&126&90&60&36&18&6\\
0&0&0&10&30&60&100&150&210&250&270&270&250&210&150&100&60&30&10\\
0&0&0&15&45&90&150&225&315&375&405&405&375&315&225&150&90&45&15\\
0&0&0&21&63&126&210&315&441&525&567&567&525&441&315&210&126&63&21\\
0&0&0&25&75&150&250&375&525&625&675&675&625&525&375&250&150&75&25\\
0&0&0&27&81&162&270&405&567&675&729&729&675&567&405&270&162&81&27\\
0&0&0&27&81&162&270&405&567&675&729&729&675&567&405&270&162&81&27\\
0&0&0&25&75&150&250&375&525&625&675&675&625&525&375&250&150&75&25\\
0&0&0&21&63&126&210&315&441&525&567&567&525&441&315&210&126&63&21\\
0&0&0&15&45&90&150&225&315&375&405&405&375&315&225&150&90&45&15\\
0&0&0&10&30&60&100&150&210&250&270&270&250&210&150&100&60&30&10\\
0&0&0&6&18&36&60&90&126&150&162&162&150&126&90&60&36&18&6\\
0&0&0&3&9&18&30&45&63&75&81&81&75&63&45&30&18&9&3\\
0&0&0&1&3&6&10&15&21&25&27&27&25&21&15&10&6&3&1
\end{bmatrix}$$

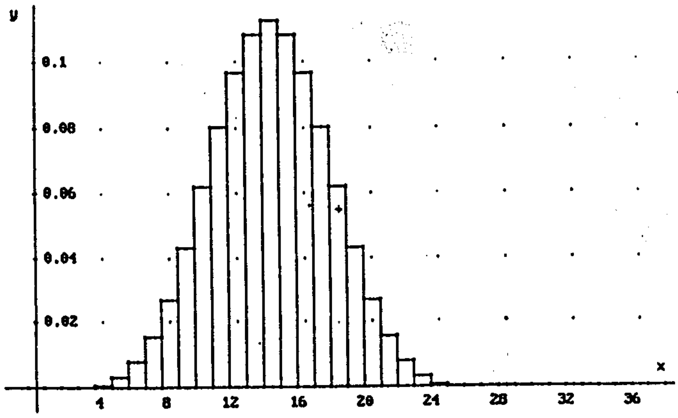
$$D6(i) := \sum_{k=MAX(0,i-18)}^{MIN(i,18)} m6_{i-k+1,k+1}$$

v6 := VECTOR(D6(k),k,0,36)

$$v6 = \frac{1}{46656} \cdot [0,0,0,0,0,0,1,6,21,56,126,252,456,756,1161,1666,2247,2856,6,3431,3906,4221,4332,4221,3906,3431,2856,2247,1666,1161,756,456,252,126,56,21,6,1]$$

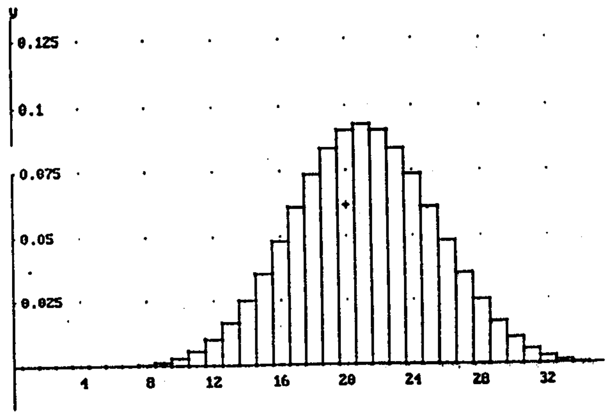
Distribution of de sum of spots of

4 di ce



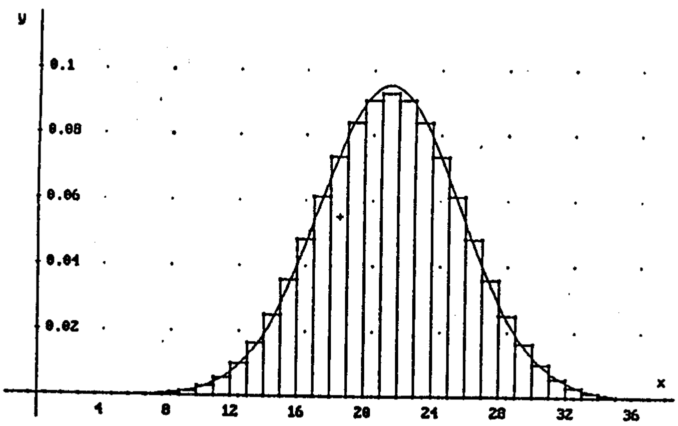
Distribution of de sum of spots of

6 di ce

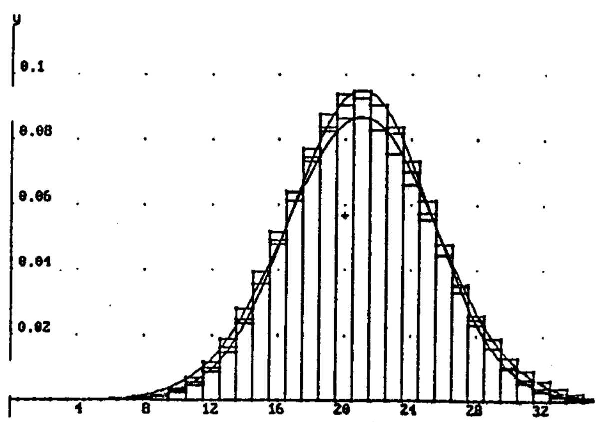


Distribution of 6 di ce and Gausscurve

with the same μ and σ



Poisson: $\mu = 21$
Binomiaal: $n = 216$ $p = 1/6$
Normaal: $\mu = 21$ $\sigma = \sqrt{105} / 6$
Normaal: $\mu = 21$ $\sigma = \sqrt{21}$
Som 6 dobbelstenen



Riemann in statistical perspective

The graphic of a density function

is the contour of

the asymptotic

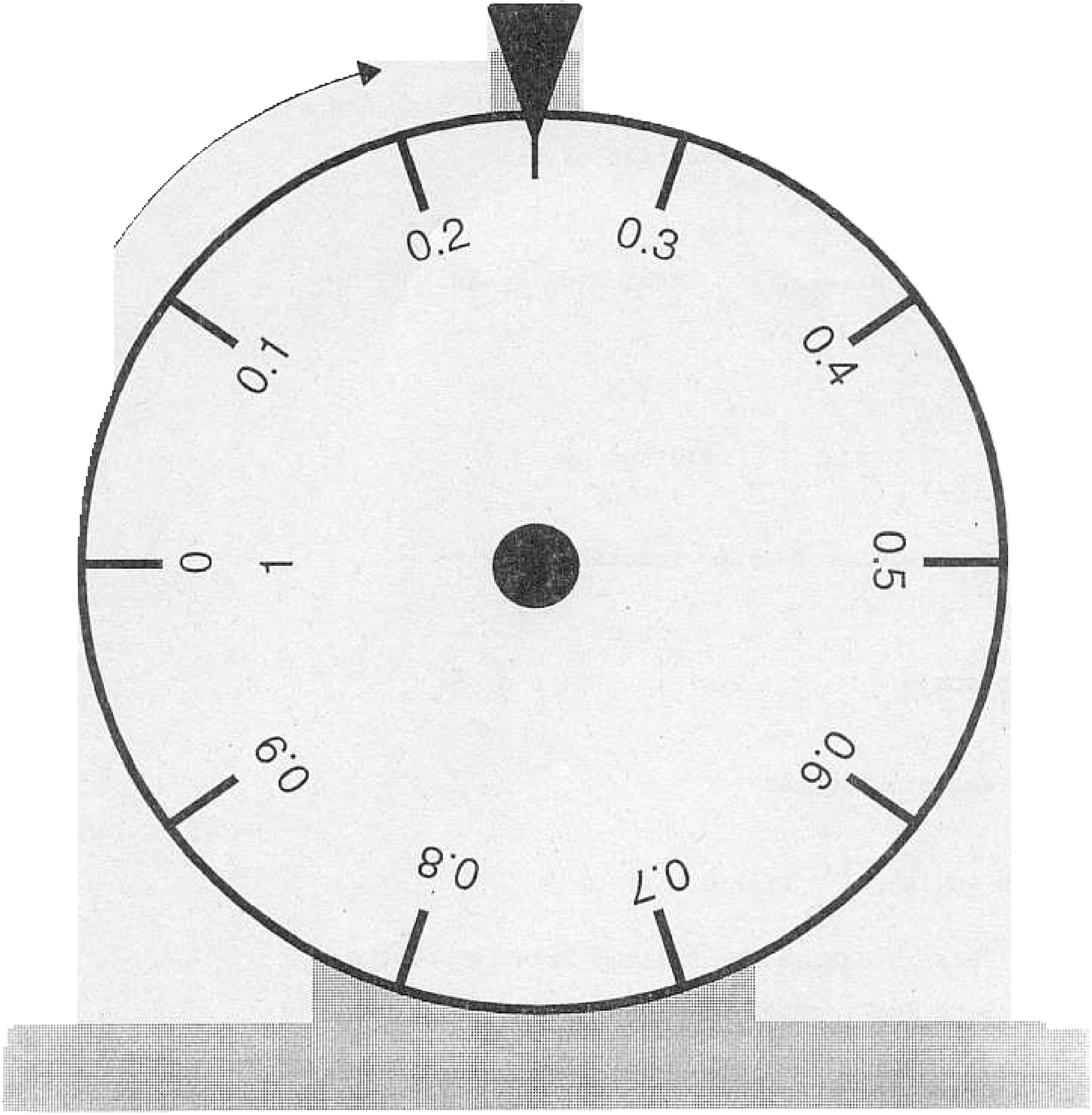
relative frequency histogram

Taking into account :

number of classes $\rightarrow \infty$

width of classes $\rightarrow 0$

Free revolving wheel



Present State: 0.246873

Convolution

"Examples for Statistics:

Y_(f,x,α):=ITERATE(f,x,α,1

"Summation of independant variables x and y"

"s = x + y , with density functions f(x) and g(y) "

"The distribution function of s:"

$$\mathrm{DIS\_SUM}(s):=\int_{-\infty}^{\infty}\int_{-\infty}^{s-x} F(x)\cdot G(y)\ dy\ dx$$

$$\int_{-\infty}^{\infty} F(x)\cdot\int_{-\infty}^{s-x} G(y)\ dy\ dx$$

"Probability density function of s:"

$$\mathrm{D\_SUM}(s):=\frac{d}{du}\int_{-\infty}^{\infty} F(x)\cdot\int_{-\infty}^{s-x} G(y)\ dy\ dx$$

"and simplified:"

$$\mathrm{D\_SUM}(s):=\int_{-\infty}^{\infty} F(x)\cdot G(s-x)\ dx$$

"This is called the Faltung Integral of f and g "

"Here implimented by:"

$$\mathrm{CONV}(f,g,x):=\int_{-\infty}^{\infty} \mathrm{Y\_}(f,x,t)\cdot \mathrm{Y\_}(g,x,x-t)\ dt$$

"(density function of sum (n times) of stochast x

DEN_SUM(n,f,x):=IF(n=1,f,CONV(DEN_SUM(n-1,f,x),f,x))

"The standard uniform distribution on [0, 1]"

U(x):=CHI(0,x,1)

$0.5\cdot\mathrm{SIGN}(x)-0.5\cdot\mathrm{SIGN}(x-1)$

CONV(U(x),U(x),x)

$$\frac{|x-2|}{2}-|x-1|+\frac{|x|}{2}$$

DEN_SUM(3,U(x),x)

$$\frac{(3-x)\cdot|x-3}{4}+\frac{3\cdot(x-2)\cdot|x-2}{4}+\frac{3\cdot(1-x)\cdot|x-1}{4}+\frac{x\cdot|x|}{4}$$

"Erwartung / Expectation / Espérance / Verwachting"

$$E(f,x):=\int_{-\infty}^{\infty} x\cdot f\,dx$$

"The standard deviation σ"

$$\sigma(f,x):=\sqrt{\left[\int_{-\infty}^{\infty} x^2\cdot f\,dx-E(f,x)^2\right]}$$

$$\sigma(U(x),x)=\frac{\sqrt{3}}{6}$$

$$UN(n,x):=\frac{n}{2}\cdot\sum_{k=0}^{n}\frac{(-1)^k\cdot\mathrm{SIGN}(x-k)\cdot(x-k)^{n-1}}{k!\cdot(n-k)!}$$

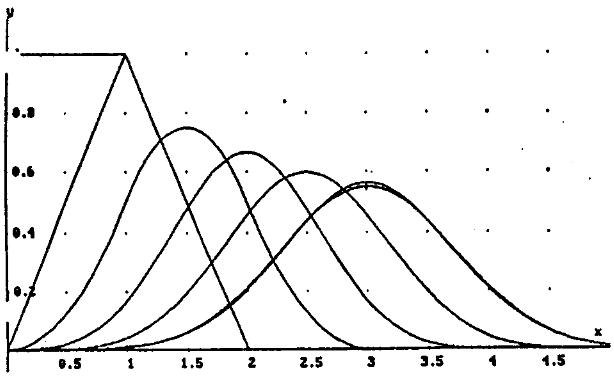
UN(3,x)

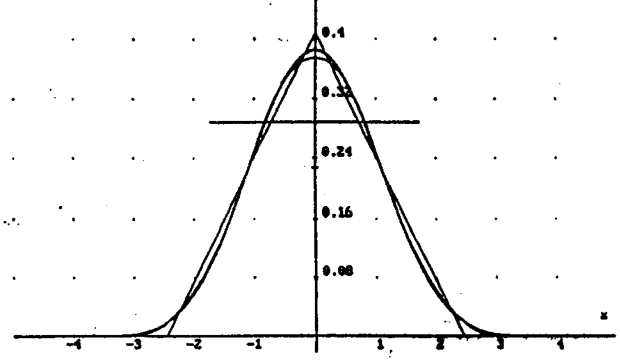
$$\frac{(3-x)\cdot|x-3|}{4}+\frac{3\cdot(x-2)\cdot|x-2|}{4}+\frac{3\cdot(1-x)\cdot|x-1}{4}+\frac{x\cdot|x|}{4}$$

E(UN(12,x),x)=6

σ(UN(12,x),x)=1

DEN_MEAN(n,f,x):=DEN_SUM(n,n·Y_(f,x,n·x),x)


y
0.8
0.6
0.4
0.2
x
0.5
1
1.5
2
2.5
3
3.5
4
4.5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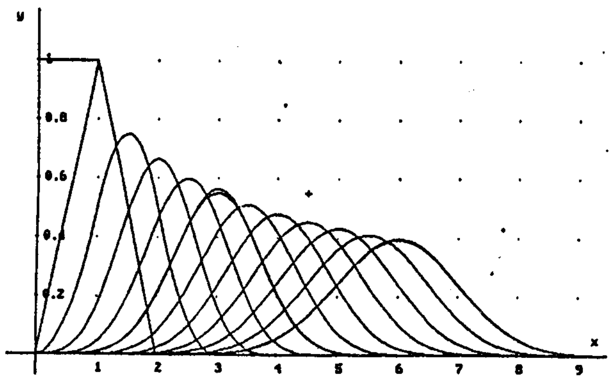
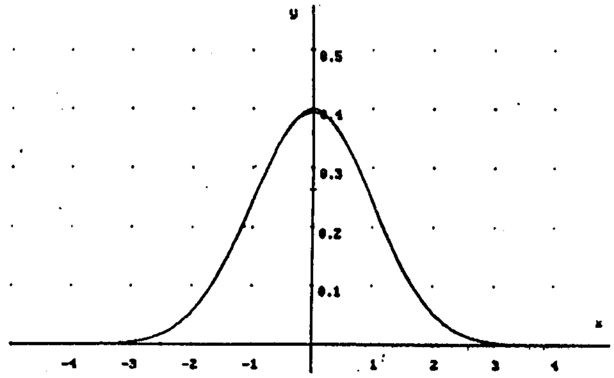
0.4
0.32
0.24
0.16
0.08
x
-4
-3
-2
-1
1
2
3
4


y
0.8
0.6
0.2
x
1
2
3
4
5
6
7
8
9


y
0.5
0.4
0.3
0.2
0.1
x
-4
-3
-2
-1
1
2
3
4

```
Derive          Days          Düsseldorf

Geben Sie bitte die Zahl n  der Ziehungen :5000
i =    250   0.043786   chisq =   2.486
i =    500  -0.556855   chisq =   0.160
i =    750   0.598560   chisq =   0.209
i =   1000   0.548003   chisq =   0.275
i =   1250   1.448830   chisq =   1.356
i =   1500  -1.164406   chisq =   1.672
j =   1750   1.127858   chisq =   3.431
. =   2000  -1.611991   chisq =   3.444
i =   2250  -0.670933   chisq =   2.865
i =   2500   2.572498   chisq =   3.487
i =   2750   1.156031   chisq =   4.736
i =   3000   1.291466   chisq =   5.800
i =   3250  -0.627515   chisq =   4.506
i =   3500   1.232472   chisq =   3.117
i =   3750  -2.347683   chisq =   2.667
i =   4000   0.118235   chisq =   2.249
i =   4250  -0.170427   chisq =   2.674
i =   4500  -1.043372   chisq =   3.046
i =   4750   2.148455   chisq =   2.064
  =   5000  -0.709319   chisq =   1.652
```

```
Derive          Days          Düsseldorf

Geben Sie bitte die Zahl n  der Ziehungen :100000
  =   5000  -0.709319   chisq =   1.652
  =  10000   0.495771   chisq =   5.607
  =  15000  -0.865236   chisq =   8.615
  =  20000  -1.184430   chisq =  13.220
  =  25000   0.708599   chisq =  18.874
  =  30000  -1.453239   chisq =  18.487
  =  35000  -0.374534   chisq =  16.777
  =  40000   0.802625   chisq =  16.139
  =  45000  -1.501352   chisq =  18.270
  =  50000   1.696444   chisq =  15.564
  =  55000  -1.058575   chisq =  14.253
  =  60000   1.341499   chisq =  18.793
  =  65000   0.567078   chisq =  17.656
  =  70000  -1.148928   chisq =  15.112
  =  75000   1.988890   chisq =  18.080
  =  80000   0.338443   chisq =  17.379
  =  85000  -2.179860   chisq =  16.874
  =  90000   1.916893   chisq =  17.030
  =  95000   0.674111   chisq =  18.801
  = 100000  -0.300295   chisq =  18.481
```

```
Derive          Days          Düsseldorf

Geben Sie bitte die Zahl n  der Ziehungen :1000000
j =   50000   1.696444   chisq =  15.564
i =  100000  -0.300295   chisq =  18.481
i =  150000  -1.355980   chisq =  25.723
i =  200000  -0.310453   chisq =  42.319
i =  250000   3.996442   chisq =  48.343
i =  300000   0.724861   chisq =  54.379
i =  350000   0.034960   chisq =  73.315
i =  400000   2.086896   chisq =  85.268
i =  450000   1.040824   chisq =  95.098
i =  500000  -0.943098   chisq = 110.008
i =  550000   0.295285   chisq = 124.548
i =  600000   0.916129   chisq = 134.275
i =  650000  -0.920409   chisq = 142.291
i =  700000   0.945828   chisq = 148.361
i =  750000   0.674994   chisq = 170.516
i =  800000   0.427247   chisq = 184.443
i =  850000   2.362743   chisq = 189.525
i =  900000   0.641638   chisq = 193.897
:    950000  -0.575912   chisq = 193.591
i - 1000000  -0.129750   chisq = 196.394
```

"The normal distribution"

$$NOR(x) := \frac{\sqrt{2}\cdot \hat{e}^{-x^2/2}}{2\cdot\sqrt{\pi}}$$

DEN_MEAN(10,NOR(x),x)

$$\frac{\sqrt{5}\cdot \hat{e}^{-5\cdot x^2}}{\sqrt{\pi}}$$

"Central limit theorem"

"This function standardizes to μ = 0 and σ = 1"

STD_MEAN(n,f,x):=$\sqrt{n}\cdot\sigma$(f,x)·Y_(DEN_SUM(n,f,x),x,$\sqrt{n}\cdot\sigma$(f,x)·x +E(f,x)·n)

STD_MEAN(3,U(x),x)

$$\frac{(3-x)\cdot|x-3}{32}+\frac{(x+3)\cdot|x+3|}{32}+\frac{3\cdot(x-1)\cdot|x-1|}{32}-\frac{3\cdot(x+1)\cdot|x+1}{32}$$

$$V(x) := \frac{|x|\cdot SIGN(x+1)}{2}-\frac{|x|\cdot SIGN(x-1)}{2}$$

"negative exponential distribution"

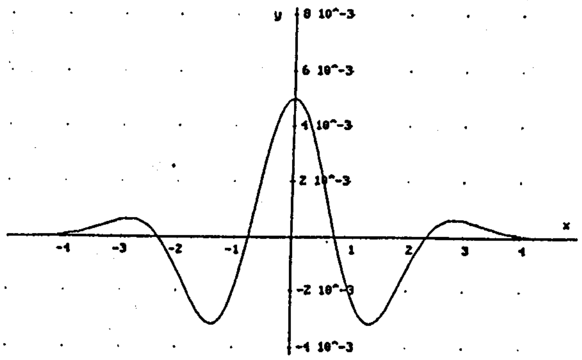
$$NEX(x) := \frac{\hat{e}^{-x}\cdot SIGN(x)}{2}+\frac{\hat{e}^{-x}}{2}$$

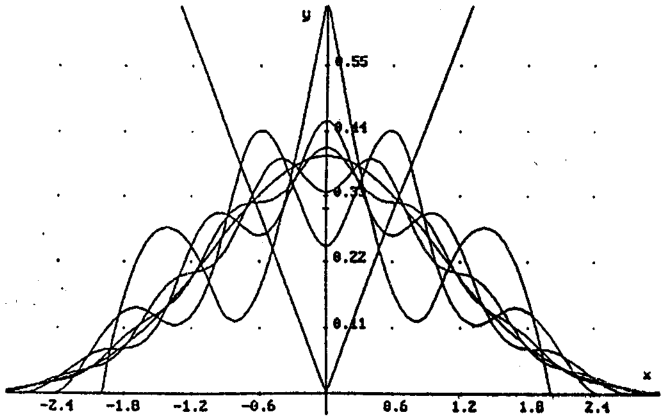
E(NEX(x),x)=1
σ(NEX(x),x)=1
DEN_SUM(2,NEX(x),x)

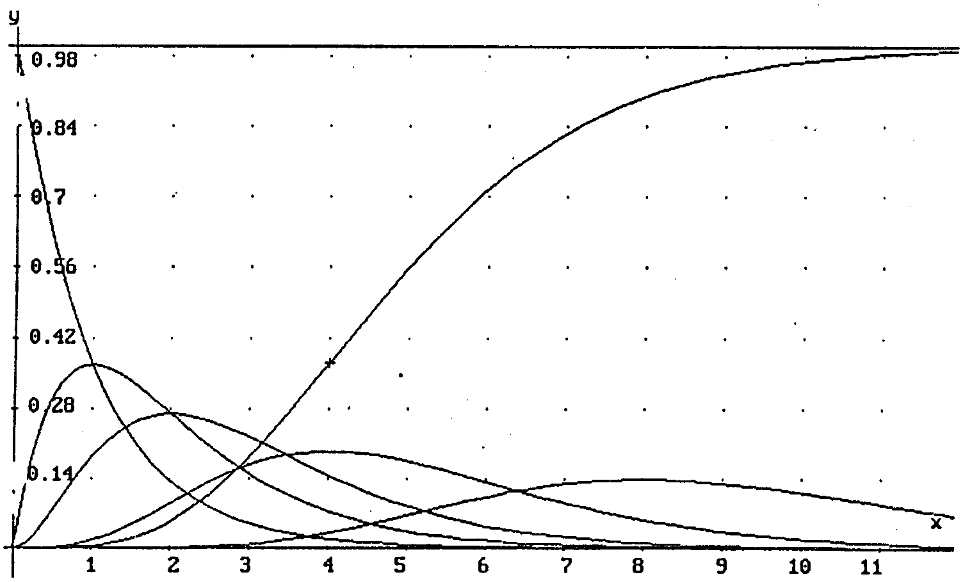
$$\frac{\hat{e}^{-x}\cdot|x|}{2}+\frac{x\cdot\hat{e}^{-x}}{2}$$

DEN_SUM(3,NEX(x),x)

$$\frac{x\cdot\hat{e}^{-x}\ \ |x|}{4}+\frac{x^2\cdot\hat{e}^{-x}}{4}$$


y
8 10^-3
6 10^-3
4 10^-3
2 10^-3
-2 10^-3
-4 10^-3
x
-4 -3 -2 -1 1 2 3 4


y
0.55
0.44
0.33
0.22
0.11
x
-2.4 -1.8 -1.2 -0.6 0.6 1.2 1.8 2.4


y
0.98
0.84
0.7
0.56
0.42
0.28
0.14
x
1 2 3 4 5 6 7 8 9 10 11

"Proof by complete induction"

#68: $\mathrm{ERL}(n,x) := \dfrac{\hat{e}^{-x} \cdot x^{n}}{2 \cdot |x| \cdot (n-1)!} + \dfrac{\hat{e}^{-x} \cdot x^{n-1}}{2 \cdot (n-1)!}$

#69: $\mathrm{ERL}(1,x) = \dfrac{\hat{e}^{-x} \cdot \mathrm{SIGN}(x)}{2} + \dfrac{\hat{e}^{-x}}{2}$

#70: "assume"

#71: $\mathrm{ERL}(n,x) = \dfrac{\hat{e}^{-x} \cdot x^{n}}{2 \cdot |x| \cdot (n-1)!} + \dfrac{\hat{e}^{-x} \cdot x^{n-1}}{2 \cdot (n-1)!}$

$\mathrm{CONV}\left[\dfrac{\hat{e}^{-x} \cdot x^{n}}{2 \cdot |x| \cdot (n-1)!} + \dfrac{\hat{e}^{-x} \cdot x^{n-1}}{2 \cdot (n-1)!}, \mathrm{NEX}(x), x\right]$

#73: $\dfrac{\hat{e}^{-x} \cdot x^{n} \cdot \mathrm{SIGN}(x)}{2 \cdot n!} + \dfrac{\hat{e}^{-x} \cdot x^{n}}{2 \cdot n!}$

#74: "Q E D"

#75: "This leads for 0 < t to:"

#76: $t :\in \mathrm{Real}\ [0, \infty)$

$\mathrm{ERL}(n,t) := \dfrac{\hat{e}^{-t} \cdot t^{n-1}}{(n-1)!}$

#78: $\mathrm{VERL}(n,t) := \dfrac{\int_0^t \hat{e}^{-x} \cdot x^{n-1}\, dx}{(n-1)!}$

#79: ERL(5,t)

#80: $\dfrac{t^{4} \cdot \hat{e}^{-t}}{24}$

#81: VERL(t,5)

```
#85: "SIMULATION"

#86: D(x):=NOR(x-6)-UN(12,x)

#90: SOLVE(D(α),α,6.74,6.76)

#91: α:=6.74982032267418

#92: SOLVE(D(ß),ß,8.33,8.35)

#93: ß:=8.33519629531484
```

$$\#94:\quad 2\cdot\int_{6}^{\alpha} G(x)\ dx$$

```
#95: PrecisionDigits:=15

#96: a:=0.54196522217911
```

$$\#97:\quad 2\cdot\int_{\alpha}^{ß} G(x)\ dx$$

```
#98: b:=0.439826987886831
#99: c:=1-a-b
#101:c=0.018207789934059
#102:e_a:=2·NORMAL(α-6)-1
#103:2·NORMAL(α-6)-1
#104:0.546637072818064
#105:e_a:=0.546637072818064
#106:2·(NORMAL(ß-6)-NORMAL(α-6)
#107:e_b:=0.433829758592318
#108:e_c:=1-e_a-e_b
#109:e_c=0.0195331168589618
```

$$\#110:q:=\frac{a^2}{e\_a}+\frac{b^2}{e\_b}+\frac{c^2}{e\_c}-1$$

```
#111:q=0.000212763902770491
```

$$\#112:2\cdot\int_{0}^{6}\frac{G(x)^2}{NOR(x-6)}\ dx-1$$

```
#113:0.00042230393246367
```